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Bayesian methodology for target tracking using combined RSS and AoA measurements

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ABSTRACT

This work addresses the target tracking problem based on received signal strength (RSS) and angle of arrival (AoA) measurements. The Bayesian methodology, which integrates the information given by observations with prior knowledge extracted from target motion model in order to enhance the estimation accuracy was employed. First, by converting the considered highly non-linear measurement model into a linear one, *i.e.*, a novel *linearization* technique of the measurement model is proposed. The derived model is then merged with the prior knowledge, and a novel maximum *a posteriori* (MAP) estimator whose solution is given in closed-form is proposed. It is also shown that the Kalman filter (KF) can be directly applied on top of the *linearized* observation model, which results in a proposal of a novel KF algorithm. Furthermore, to the best of authors' knowledge, this paper premierly presents the application of the extended KF (EKF) and the unscented KF (UKF) to the considered tracking problem, by applying first-order *linearization* technique to the original non-linear model, and by applying the unscented transformation to carefully selected sample points, respectively. Finally, importance weights are computed for a large number of randomly selected sample points to render a well-known particle filter (PF) solution. Simulation results show that the proposed algorithms perform better than a *naive* one which uses only information from observations. They also confirm the effectiveness of the proposed *linearization* technique in comparison with the existing one, reducing the estimation error for about 25%.

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1. Introduction

Accurate localization of a moving target has become an important task because of its relevance in both military and commercial applications [1–5]. Cost, complexity and applicability in all terrain, limit the use of the existing location aware systems (*e.g.*, GPS). Hence, new techniques to enhance accuracy and reduce the cost have been proposed by the researchers. Amongst other, these include localization algorithms based on received signal strength (RSS), time of arrival, angle of arrival (AoA) or a combination of them [5–15].

The work described in [7–15] considered the *classical* target localization problem, based on observations exclusively. In [2,3]

and [5], the authors considered target tracking problem, where the observations were combined with some prior knowledge to enhance the estimation accuracy. However, they considered RSS-based target tracking problem only. The inventors of [6] tackled the target tracking problem by making use of radiated signal strength and the relatively measured radio carrier phase-delay to detect and position the radio transmitting device(s). In [4], the authors investigated the target tracking problem by employing hybrid, RSS and AoA, measurements. The authors first *linearized* the highly non-linear measurement model and on top of the *linearized* model they applied a Kalman filter (KF). In [4], the measurement model was *linearized* by using a very simple and intuitive approach. It can be summarized as forming a line and using the RSS measurements to determine the length of the line. At one point of the line, the authors situated a known anchor location and used the AoA to determine the slope of the line. In that way, an estimate of the target location was obtained at the other point of the line. Although this is an effective way to tackle the non-linearity of the measurement model, the authors in [4] treated all links as equal,

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and no mitigation technique was used to deal with potentially negative impact from distant links. Besides the KF, a particle filter (PF) algorithm was also proposed in [4], as well as a generalized pattern search method for estimating the path loss exponent (PLE) for each link in every time step.

In this work, the target tracking problem by taking advantage of coupled RSS and AoA measurements is addressed. Various algorithms that are based on the Bayesian approach, which integrates the information gathered through radio observations with prior knowledge given by target state transition model are proposed. Therefore, the main contributions of this work are as follows. First, a novel *linearization* technique to efficiently represent the original non-linear measurement model with a linear one is proposed. The proposed *linearization* technique is fundamentally different from the existing one described in [4], since a Cartesian to polar (spherical for 3-dimensional space) coordinates conversion to deal effortlessly with the non-linear terms in the measurement model is used here. Moreover, in contrast to [4] where the authors treat all links as equal, here, weights which give more importance to *nearby* links and mitigate the potential negative impact of the remote ones are applied. This *linearized* measurement model is then combined with the prior knowledge to design a novel maximum *a posteriori* (MAP) estimator. Owing to the use of the derived measurement model, it is also shown that the application of the KF is straightforward, and thus, a novel KF algorithm is proposed. Although various extensions of the KF are available in the literature nowadays, to the best of authors' knowledge, none of them were applied to the considered RSS-AoA-based target tracking problem. Therefore, by applying first-order Taylor series expansion, the extended KF (EKF) is developed also. Furthermore, to capture the mean and covariance up to a higher order of Taylor series expansion, a number of carefully chosen sample points were used with the unscented transformation to build an unscented KF (UKF). Finally, a large number of sample points were chosen at random and importance weights were assigned to them by calculating the likelihood that these points correspond to a given observation in order to derive a PF.

This paper is organized as follows. In Section 2, we introduce the target state transition model as well as the measurement model, and we formulate the target tracking problem by using the Bayesian approach. Section 3 describes the proposed technique used to *linearize* the measurement model. Section 4 present the derivation of the proposed tracking algorithms. In Section 5, simulation results are presented for two different target trajectories in order to validate the performance of the proposed algorithms. Finally, Section 6 summarizes the main conclusions.

2. Problem formulation

Let $\mathbf{x}_t = [x_x, x_y]^T$ and $\mathbf{a}_i = [a_{ix}, a_{iy}]^T$, for $i = 1, \dots, N$ denote the unknown location of a moving target at time t and known location of the i th static anchor, respectively¹. For simplicity, a constant velocity motion model is assumed here, perturbed only by slight speed corrections, so that the velocity components in the x and y directions at time t are given by

$$\mathbf{v}_t = \mathbf{v}_{t-1} + \mathbf{r}_{v_t}, \quad (1)$$

where $\mathbf{r}_{v_t}$ represents the noise perturbations. Hence, from the equations of motion [16], the target location at time t is

$$\mathbf{x}_t = \mathbf{x}_{t-1} + \mathbf{v}_{t-1}\Delta + \mathbf{r}_{x_t}, \quad (2)$$

where Δ and $\mathbf{r}_{x_t}$ are the sampling interval between two consecutive time steps and location process noise, respectively. Now, if

the target state at t is described by its location and velocity, i.e., $\boldsymbol{\theta}_t = [\mathbf{x}_t^T, \mathbf{v}_t^T]^T$, from (1) and (2) we get

$$\boldsymbol{\theta}_t = \mathbf{S}\boldsymbol{\theta}_{t-1} + \mathbf{r}_t, \quad (3)$$

where $\mathbf{r}_t = [\mathbf{r}_{x_t}^T, \mathbf{r}_{v_t}^T]^T$ is the state process noise [1–5]. This process noise is assumed to be zero-mean Gaussian with a covariance matrix $\mathbf{Q}$, i.e., $\mathbf{r}_t \sim \mathcal{N}(\mathbf{0}, \mathbf{Q})$, where $\mathbf{Q}$ is defined as

$$\mathbf{Q} = q \begin{bmatrix} \frac{\Delta^3}{3} & 0 & \frac{\Delta^2}{2} & 0 \\ 0 & \frac{\Delta^3}{3} & 0 & \frac{\Delta^2}{2} \\ \frac{\Delta^2}{2} & 0 & \Delta & 0 \\ 0 & \frac{\Delta^2}{2} & 0 & \Delta \end{bmatrix},$$

with q denoting the state process noise intensity [1,3,17]. The symbol $\mathbf{S}$ in (3) stands for the state transition matrix, given by

$$\mathbf{S} = \begin{bmatrix} 1 & 0 & \Delta & 0 \\ 0 & 1 & 0 & \Delta \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

At each time step, the target broadcasts a signal to anchors which then extract the RSS and AoA information from it. Hence, the measurement equation can be expressed as

$$\mathbf{z}_t = \mathbf{h}(\mathbf{x}_t) + \mathbf{n}_t, \quad (4)$$

where $\mathbf{z}_t = [\mathbf{P}_t^T, \boldsymbol{\phi}_t^T]^T$ ($\mathbf{z}_t \in \mathbb{R}^{2N}$) is the observation vector composed of RSS, $\mathbf{P}_t = [P_i^{(t)}]^T$, and AoA, $\boldsymbol{\phi}_t = [\phi_i^{(t)}]^T$, measurements at time t . The function $\mathbf{h}(\mathbf{x}_t)$ in (4) is defined as $h_i(\mathbf{x}_t) = P_0 - 10\gamma \log_{10} \frac{d_i}{d_0}$ for $i = 1, \dots, N$ [18], where P_0 (dBm) is the reference power at a distance d_0 ($d_0 \leq d_i$), γ is the path loss exponent (PLE) and d_i is the distance between the target and the i th anchor, and $h_i(\mathbf{x}_t) = \tan^{-1} \left(\frac{x_y - a_{iy}}{x_x - a_{ix}} \right)$ for $i = N+1, \dots, 2N$ [19]. The measurement noise is modeled as $\mathbf{n}_t \sim \mathcal{N}(\mathbf{0}, \mathbf{C})$, where the noise covariance is defined as $\mathbf{C} = \text{diag}([\sigma_{n_i}^2, \sigma_{m_i}^2]) \otimes \mathbf{I}_4$, with $\sigma_{n_i}^2$ (dB) and $\sigma_{m_i}^2$ (rad) being the variances of the RSS and AoA measurement noise, respectively, $\mathbf{I}_M$ representing the identity matrix of size M and symbol $\otimes$ denoting the Kronecker product.

Our goal is to incorporate the prior knowledge, given by the state transition model (3) to construct the estimation of the state at the next time instant, into our estimator. This approach is known as the Bayesian approach [16], and it can improve the estimation accuracy in comparison with the *classical approach* which disregards the prior knowledge. By using (3) and (4), we can build the marginal posterior probability distribution function (PDF), $p(\boldsymbol{\theta}_t | \mathbf{z}_{1:t})$, to quantify the belief we have in the values of the state $\boldsymbol{\theta}_t$ given all the past measurements $\mathbf{z}_{1:t}$. From $p(\boldsymbol{\theta}_t | \mathbf{z}_{1:t})$, we can obtain an estimate at any time step we desire.

Below, a recursive approach for the evaluation of $p(\boldsymbol{\theta}_t | \mathbf{z}_{1:t})$ at any time instant is presented, which is typical for Bayesian methods [1–5].

- *Initialization*: The marginal posterior PDF at $t = 0$ is set to the prior PDF $p(\boldsymbol{\theta}_0)$ of $\boldsymbol{\theta}_0$.
- *Prediction*: By using the state transition model (3), the predictive PDF of the state at t is given by

$$p(\boldsymbol{\theta}_t | \mathbf{z}_{1:t-1}) = \int p(\boldsymbol{\theta}_t | \boldsymbol{\theta}_{t-1}) p(\boldsymbol{\theta}_{t-1} | \mathbf{z}_{1:t-1}) d\boldsymbol{\theta}_{t-1}. \quad (5)$$

- *Update*: By following the Bayes' rule [1,17], we have

$$p(\boldsymbol{\theta}_t | \mathbf{z}_{1:t}) = \frac{p(\mathbf{z}_t | \boldsymbol{\theta}_t) p(\boldsymbol{\theta}_t | \mathbf{z}_{1:t-1})}{\int p(\mathbf{z}_t | \boldsymbol{\theta}_t) p(\boldsymbol{\theta}_t | \mathbf{z}_{1:t-1}) d\boldsymbol{\theta}_t}. \quad (6)$$

¹ Without loss of generality, this paper focuses on 2-dimensional scenario. The extension to 3-dimensional scenario is straightforward.

Note that the denominator in (6) is just a normalizing constant. In general, the marginal PDF at $t - 1$ cannot be calculated analytically, and the integral in (5) cannot be obtained analytically if the state model is non-linear. Therefore, some approximations are required in order to obtain $p(\theta_t | \mathbf{z}_{1:t})$.

3. Linearization of the observation model

Assuming that the noise term is sufficiently small, from (4) it can be written for the RSS measurement model

$$\lambda_i \|\mathbf{x}_t - \mathbf{a}_i\| \approx \eta d_0, \text{ for } i = 1, \dots, N, \quad (7)$$

where $\lambda_i = 10^{\frac{p_i^{(t)}}{10\gamma}}$, $\eta = 10^{\frac{p_0}{10\gamma}}$. According to Cartesian to polar coordinates conversion, one can express $\mathbf{x}_t - \mathbf{a}_i = r_i \mathbf{u}_i : r_i \geq 0, \|\mathbf{u}_i\| = 1$, where the unit vector can be found by using the available AoA information, i.e., $\mathbf{u}_i = [\cos(\phi_i^{(t)}), \sin(\phi_i^{(t)})]^T$. Applying this conversion in (7) and multiplying the left hand side of (7) by $\mathbf{u}_i^T \mathbf{u}_i$, yields

$$\lambda_i \mathbf{u}_i^T (\mathbf{x} - \mathbf{a}_i) \approx \eta d_0 \quad (8)$$

From (4), similar can be done for the AoA measurement model

$$\mathbf{c}_i^T (\mathbf{x} - \mathbf{a}_i) \approx 0, \text{ for } i = N + 1, \dots, 2N, \quad (9)$$

where $\mathbf{c}_i = [-\sin(\phi_i^{(t)}), \cos(\phi_i^{(t)})]^T$.

By introducing weights, $\mathbf{w} = [w_i]^T$, in (8) and (9) such that $w_i = 1 - \frac{\hat{d}_i}{\sum_{i=1}^N \hat{d}_i}$ and $\hat{d}_i = d_0 10^{\frac{p_0 - p_i^{(t)}}{10\gamma}}$, in order that more importance is given to *nearby* links, yields respectively

$$w_i \lambda_i \mathbf{u}_i^T (\mathbf{x}_t - \mathbf{a}_i) \approx w_i \eta d_0, \text{ for } i = 1, \dots, N, \quad (10a)$$

$$w_i \mathbf{c}_i^T (\mathbf{x}_t - \mathbf{a}_i) \approx 0, \text{ for } i = N + 1, \dots, 2N. \quad (10b)$$

Thus, the RSS and AoA measurement models can be *linearized* by (10) and written in a vector form as a function of the target state as

$$\mathbf{A}\theta_t = \mathbf{b}. \quad (11)$$

where

$$\mathbf{A} = \begin{bmatrix} \vdots & \vdots & \vdots \\ w_i \lambda_i \mathbf{u}_i^T & 0 & 0 \\ \vdots & \vdots & \vdots \\ w_i \mathbf{c}_i^T & 0 & 0 \\ \vdots & \vdots & \vdots \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} \vdots \\ w_i (\lambda_i \mathbf{u}_i^T \mathbf{a}_i + \eta d_0) \\ \vdots \\ w_i \mathbf{c}_i^T \mathbf{a}_i \\ \vdots \end{bmatrix},$$

By applying the least squares (LS) principle to (11), a target state estimate can be obtained as

$$\hat{\theta}_{t|t} = \arg \min_{\theta_t} \|\mathbf{A}\theta_t - \mathbf{b}\|^2, \quad (12)$$

whole solution is readily found as $\hat{\theta}_{t|t} = (\mathbf{A}^T \mathbf{A})^{-1} (\mathbf{A}^T \mathbf{b})$.

In order to show that by employing the proposed *linearization* technique, one gets a tight approximation of the maximum likelihood (ML) estimator, we call the reader's attention to Fig. 1. In this figure, a possible realization of the ML and the LS in (12) objective functions are plotted, when all sensors were randomly deployed in a square region with a border of $B = 20$ m. The true target location was set at $[5.09; 2.11]$ and $N = 5$ anchors were deployed at $[6.36, 18.80, 9.59, 10.89, 10.88, 10.45; 2.38, 12.91, 12.79, 12.95, 14.42, 19.87]$, and the rest of the parameters are set as in Section 5. Fig. 1 shows that the optimal solutions of the two functions are close to each other, meaning that the

Fig. 1. Illustration of the objective functions versus x (m) and y (m) coordinates, when $N = 5$, $\sigma_{n_i} = 9$ dB, $\sigma_{m_i} = 4$ degrees, $\gamma = 3$, $\gamma_i \sim \mathcal{U}[2.7, 3.3]$, $P_0 = -10$ dBm; the minimum of the objective function is indicated by a white square.

LS objective function (i.e., the new *linearization* technique) is an excellent approximation of the ML one. Please note that it is so, even for the case when the assumption that the noise power is low does not hold. However, the objective function in (12) is much smoother than the ML one, which allows us to easily obtain its global minimum.

4. The proposed methods

4.1. MAP-based estimator

From $p(\theta_t | \mathbf{z}_{1:t})$, a state estimate, $\hat{\theta}_{t|t}$, of θ_t can be obtained according to the maximum *a posteriori* (MAP) criteria [16], by maximizing the marginal posterior PDF, i.e.,

$$\hat{\theta}_{t|t} = \arg \max_{\theta_t} p(\theta_t | \mathbf{z}_{1:t}) \approx \arg \max_{\theta_t} p(\mathbf{z}_t | \theta_t) p(\theta_t | \mathbf{z}_{1:t-1}). \quad (13)$$

This is reminiscent of the ML estimator, except for the existence of the prior PDF. The problem in (13) is highly non-convex and its analytical solution cannot be obtained in general. Thus, (13) is approximated by another estimator whose solution is readily obtained.

First, assume that $p(\theta_{t-1} | \mathbf{z}_{1:t-1})$ has Gaussian distribution with mean $\hat{\theta}_{t-1|t-1}$ and covariance $\hat{\mathbf{P}}_{t-1|t-1}$ (a similar assumption was made in [17]). According to (5), we obtain

$$p(\theta_t | \mathbf{z}_{1:t-1}) \approx \frac{1}{k_1} \exp \left\{ -\frac{1}{2} (\theta_t - \hat{\theta}_{t|t-1})^T \hat{\mathbf{P}}_{t|t-1}^{-1} (\theta_t - \hat{\theta}_{t|t-1}) \right\}, \quad (14)$$

where k_1 is a constant, and $\hat{\theta}_{t|t-1}$ and $\hat{\mathbf{P}}_{t|t-1}$ are the mean and the covariance of the one-step predicted state, respectively, obtained through (3) as

$$\begin{aligned} \hat{\theta}_{t|t-1} &= \mathbf{S} \hat{\theta}_{t-1|t-1}, \\ \hat{\mathbf{P}}_{t|t-1} &= \mathbf{S} \hat{\mathbf{P}}_{t-1|t-1} \mathbf{S}^T + \mathbf{Q}. \end{aligned} \quad (15)$$

The likelihood function can be written as

$$p(\mathbf{z}_t | \boldsymbol{\theta}_t) = \frac{1}{k_2} \exp \left(-\frac{1}{2} (\mathbf{z}_t - \mathbf{h}(\mathbf{x}_t))^T \mathbf{C}^{-1} (\mathbf{z}_t - \mathbf{h}(\mathbf{x}_t)) \right), \quad (16)$$

where k_2 is a constant. In Section 3, it was shown how to tightly approximate (16) by another, linear estimator. Therefore, (13) can be written as

$$\hat{\boldsymbol{\theta}}_{t|t} = \arg \min_{\boldsymbol{\theta}_t} (\mathbf{z}_t - \mathbf{h}(\mathbf{x}_t))^T \mathbf{C}^{-1} (\mathbf{z}_t - \mathbf{h}(\mathbf{x}_t)) + (\boldsymbol{\theta}_t - \hat{\boldsymbol{\theta}}_{t|t-1})^T \hat{\mathbf{P}}_{t|t-1}^{-1} (\boldsymbol{\theta}_t - \hat{\boldsymbol{\theta}}_{t|t-1}). \quad (17)$$

By following a similar approach as in (12), the problem (17) can be approximated by

$$\hat{\boldsymbol{\theta}}_{t|t} = \arg \min_{\boldsymbol{\theta}_t} \|\tilde{\mathbf{A}}\boldsymbol{\theta}_t - \tilde{\mathbf{b}}\|^2, \quad (18)$$

whose solution is readily given by

$$\hat{\boldsymbol{\theta}}_{t|t} = (\tilde{\mathbf{A}}^T \tilde{\mathbf{A}})^{-1} (\tilde{\mathbf{A}}^T \tilde{\mathbf{b}}), \quad (19)$$

where $\tilde{\mathbf{A}} = [\mathbf{A}; \hat{\mathbf{P}}_{t|t-1}^{-\frac{1}{2}}]$ and $\tilde{\mathbf{b}} = [\mathbf{b}; \hat{\mathbf{P}}_{t|t-1}^{-\frac{1}{2}} \hat{\boldsymbol{\theta}}_{t|t-1}]$.

The step by step proposed MAP-based algorithm² is outlined in Algorithm 1.

Algorithm 1 MAP Algorithm Description

Require: $\mathbf{z}_t$, for $t = 0, \dots, T-1, \mathbf{Q}, \mathbf{S}$

```

1: Initialization:  $\hat{\boldsymbol{\theta}}_{0|0} \leftarrow (12)$ ,  $\hat{\mathbf{P}}_{0|0} \leftarrow \mathbf{I}_4$  for  $t = 0, \dots, T-1$ 
2: for  $t = 1, \dots, T-1$  do
3:   Prediction:
4:      $\hat{\boldsymbol{\theta}}_{t|t-1} \leftarrow (15)$ 
5:      $\hat{\mathbf{P}}_{t|t-1} \leftarrow (19)$ 
6:   Update:
7:      $\hat{\boldsymbol{\theta}}_{t|t} \leftarrow (19)$ 
8: end for
```

4.2. Kalman filter

When both the state and observation models are linear and the noise is assumed to be zero-mean with finite covariance, the KF provides the optimal solution in the least squares sense [1]. Although the observation model (4) is non-linear, it can be *linearized* according to (11). Then, the mean and the covariance are updated according to the KF [16], i.e.,

$$\begin{aligned} \hat{\boldsymbol{\theta}}_{t|t} &= \hat{\boldsymbol{\theta}}_{t|t-1} + \mathbf{K}_t (\mathbf{b} - \mathbf{A} \hat{\boldsymbol{\theta}}_{t|t-1}), \\ \hat{\mathbf{P}}_{t|t} &= (\mathbf{I}_4 - \mathbf{K}_t \mathbf{A}) \hat{\mathbf{P}}_{t|t-1}, \end{aligned} \quad (20)$$

where $\mathbf{K}_t$ is the Kalman gain at t .

The step by step proposed KF algorithm³ is outlined in Algorithm 2.

4.3. Extended Kalman filter

The EKF requires no assumptions about the linearity of the state or observation models. Instead, it approximates the non-linear

² Note that an update of the state covariance matrix is not considered here. Even though this could have been done fairly easily by taking advantage of the solution obtained by solving (19) and applying Karush–Kuhn–Tucker conditions to get an update on the state covariance (similar as in [17]), our simulations showed that this approach offered only marginal improvement for the MAP estimator; thus, this approach was not applied here.

³ It is worth noting that the KF estimator could also be derived directly from (17) by simply setting the derivative of (17) to zero and then calculating the updated covariance [16] as: $\hat{\mathbf{P}}_{t|t} = \mathbb{E}[\hat{\boldsymbol{\theta}}_{t|t} \hat{\boldsymbol{\theta}}_{t|t}^T]$, at each time step t . However, to do so, one would have to assume the measurement noise covariance perfectly known. Since this assumption might not hold in practice, and the measurement model is *linearized*, a different estimator whose closed-form solution is readily available (19) was derived instead.

Algorithm 2 KF Algorithm Description

Require: $\mathbf{z}_t$, for $t = 0, \dots, T-1, \mathbf{Q}, \mathbf{C}, \mathbf{S}$

```

1: Initialization:  $\hat{\boldsymbol{\theta}}_{0|0} \leftarrow (12)$ ,  $\hat{\mathbf{P}}_{0|0} \leftarrow \mathbf{I}_4$ 
2: for  $t = 1, \dots, T-1$  do
3:   Prediction:
4:      $\hat{\boldsymbol{\theta}}_{t|t-1} \leftarrow (15)$ 
5:      $\hat{\mathbf{P}}_{t|t-1} \leftarrow (19)$ 
6:   Update:
7:      $\mathbf{K}_t \leftarrow \hat{\mathbf{P}}_{t|t-1} \mathbf{A}^T (\mathbf{A} \hat{\mathbf{P}}_{t|t-1} \mathbf{A}^T + \mathbf{C})^{-1}$ 
8:      $\hat{\boldsymbol{\theta}}_{t|t} \leftarrow (20)$ 
9:      $\hat{\mathbf{P}}_{t|t} \leftarrow (20)$ 
10: end for
```

models by their first order Taylor series expansion⁴ [16]. Therefore, the state and state covariance update for the EKF is given by

$$\begin{aligned} \hat{\boldsymbol{\theta}}_{t|t} &= \hat{\boldsymbol{\theta}}_{t|t-1} + \mathbf{K}_t (\mathbf{z}_t - \mathbf{h}(\mathbf{x} | \hat{\mathbf{x}}_{t|t-1})), \\ \hat{\mathbf{P}}_{t|t} &= (\mathbf{I}_4 - \mathbf{K}_t \mathbf{J}_{\mathbf{h}(\boldsymbol{\theta}) | \hat{\mathbf{x}}_{t|t-1}}) \hat{\mathbf{P}}_{t|t-1}, \end{aligned} \quad (21)$$

where $\hat{\mathbf{x}}_{t|t-1}$ is the one-step predicted target location, $\mathbf{h}(\mathbf{x} | \hat{\mathbf{x}}_{t|t-1})$ is the $\mathbf{h}(\mathbf{x})$ evaluated at $\hat{\mathbf{x}}_{t|t-1}$, and $\mathbf{J}_{\mathbf{h}(\boldsymbol{\theta}) | \hat{\mathbf{x}}_{t|t-1}} \in \mathbb{R}^{2N \times 4}$ is the Jacobian of $\mathbf{h}(\boldsymbol{\theta})$ evaluated at $\hat{\mathbf{x}}_{t|t-1}$, i.e.,

$$\mathbf{J}_{\mathbf{h}(\boldsymbol{\theta}) | \hat{\mathbf{x}}_{t|t-1}} = \begin{bmatrix} \vdots & \vdots & \vdots & \vdots \\ \xi \frac{\hat{x}_{x,t|t-1} - a_{xi}}{\|\hat{\mathbf{x}}_{t|t-1} - \mathbf{a}_i\|^2} & \xi \frac{\hat{x}_{y,t|t-1} - a_{yi}}{\|\hat{\mathbf{x}}_{t|t-1} - \mathbf{a}_i\|^2} & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ -\xi \frac{\hat{x}_{y,t|t-1} - a_{yi}}{\|\hat{\mathbf{x}}_{t|t-1} - \mathbf{a}_i\|^2} & \xi \frac{\hat{x}_{x,t|t-1} - a_{xi}}{\|\hat{\mathbf{x}}_{t|t-1} - \mathbf{a}_i\|^2} & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix},$$

with $\xi = \frac{10d_0\gamma}{\ln(10)}$. For the sake of simplicity, we denote $\mathbf{J}_{\mathbf{h}(\boldsymbol{\theta}) | \hat{\mathbf{x}}_{t|t-1}}$ as $\mathbf{J}$.

The step by step proposed EKF algorithm is outlined in Algorithm 3.

Algorithm 3 EKF Algorithm Description

Require: $\mathbf{z}_t$, for $t = 0, \dots, T-1, \mathbf{Q}, \mathbf{C}, \mathbf{S}$

```

1: Initialization:  $\hat{\boldsymbol{\theta}}_{0|0} \leftarrow (12)$ ,  $\hat{\mathbf{P}}_{0|0} \leftarrow \mathbf{I}_4$ 
2: for  $t = 1, \dots, T-1$  do
3:   Prediction:
4:      $\hat{\boldsymbol{\theta}}_{t|t-1} \leftarrow (15)$ 
5:      $\hat{\mathbf{P}}_{t|t-1} \leftarrow (19)$ 
6:   Update:
7:      $\mathbf{K}_t \leftarrow \hat{\mathbf{P}}_{t|t-1} \mathbf{J}^T (\mathbf{J} \hat{\mathbf{P}}_{t|t-1} \mathbf{J}^T + \mathbf{C})^{-1}$ 
8:      $\hat{\boldsymbol{\theta}}_{t|t} \leftarrow (21)$ 
9:      $\hat{\mathbf{P}}_{t|t} \leftarrow (21)$ 
10: end for
```

4.4. Unscented Kalman filter

The UKF utilizes the unscented transformation, which is founded on the intuition that *it is easier to approximate a probability distribution than it is to approximate an arbitrary non-linear function or transformation* [20]. The basic idea is to represent the state distribution by a random variable, which is described by a minimal set of carefully selected points, called sigma points. These points completely capture the mean and covariance of the random variable, and when passed through the non-linear system, they capture the mean and covariance up to the 3rd order (Taylor series expansion) for any non-linearity [20]. The sigma points, χ_i , and

⁴ However, since a linear transition model (3) is considered here, no approximation of the state model is required.

their corresponding weights, w_i , at t are given by

$$\begin{aligned} \chi_0 &= \hat{\theta}_{t-1|t-1}, \\ \chi_i &= \chi_0 + \left(\sqrt{L + \kappa \hat{\mathbf{P}}_{t-1|t-1}} \right)_i, \quad i = 1, \dots, L, \\ \chi_i &= \chi_0 - \left(\sqrt{L + \kappa \hat{\mathbf{P}}_{t-1|t-1}} \right)_i, \quad i = L + 1, \dots, 2L, \\ w_0^{(m)} &= \frac{\kappa}{\kappa + L}, \\ w_0^{(c)} &= w_0^{(m)} + (1 - \varrho^2 + \beta), \\ w_i^{(m)} &= w_i^{(c)} = \frac{1}{2(\kappa + L)}, \quad i = 1, \dots, 2L, \end{aligned} \quad (22)$$

where $\kappa = \varrho^2(L + \zeta) - L$ is a scaling parameter, L is the size of $\hat{\theta}_{t-1|t-1}$, $\varrho = 10^{-3}$ determines the spread of sigma points around the mean, $\zeta = 0$ is a secondary scaling parameter, $\beta = 2$ is used to incorporate prior knowledge of the distribution of θ , and $(\sqrt{L + \kappa \hat{\mathbf{P}}_{t-1|t-1}})_i$ is the i th column of the matrix square root. These sigma points are then propagated through the system model, i.e.,

$$\hat{\chi}_i = \mathbf{S} \chi_i, \quad i = 0, \dots, 2L. \quad (23)$$

The predicted mean and covariance are obtained as [20–24]

$$\begin{aligned} \hat{\theta}_{t|t-1} &= \sum_{i=0}^{2L} w_i^{(m)} \hat{\chi}_i, \\ \hat{\mathbf{P}}_{t|t-1} &= \sum_{i=0}^{2L} w_i^{(c)} (\hat{\chi}_i - \hat{\theta}_{t|t-1})(\hat{\chi}_i - \hat{\theta}_{t|t-1})^T + \mathbf{Q}. \end{aligned} \quad (24)$$

Similar is done with the measurement model. First, the predicted sigma points are passed through the observation model, and the predicted observation, innovation covariance and cross covariance are respectively computed as

$$\begin{aligned} \hat{\mathbf{z}}_{t|t-1} &= \sum_{i=0}^{2L} w_i^{(m)} h(\mathbf{x}|\hat{\chi}_i), \\ \hat{\mathbf{P}}_t^{\text{ZZ}} &= \sum_{i=0}^{2L} w_i^{(c)} (h(\mathbf{x}|\hat{\chi}_i) - \hat{\mathbf{z}}_{t|t-1})(h(\mathbf{x}|\hat{\chi}_i) - \hat{\mathbf{z}}_{t|t-1})^T + \mathbf{C}, \\ \hat{\mathbf{P}}_t^{\Theta Z} &= \sum_{i=0}^{2L} w_i^{(c)} (\hat{\chi}_i - \hat{\theta}_{t|t-1})(h(\mathbf{x}|\hat{\chi}_i) - \hat{\mathbf{z}}_{t|t-1})^T. \end{aligned} \quad (25)$$

Finally, the state and covariance update is obtained as

$$\begin{aligned} \hat{\theta}_{t|t} &= \hat{\theta}_{t|t-1} + \mathbf{K}_t(\mathbf{z}_t - \hat{\mathbf{z}}_{t|t-1}), \\ \hat{\mathbf{P}}_{t|t} &= \hat{\mathbf{P}}_{t|t-1} - \mathbf{K}_t \hat{\mathbf{P}}_t^{\text{ZZ}} \mathbf{K}_t^T. \end{aligned} \quad (26)$$

The step by step proposed UKF algorithm is outlined in Algorithm 4.

Algorithm 4 UKF Algorithm Description

Require: $\mathbf{z}_t$, for $t = 0, \dots, T - 1$, $\mathbf{Q}$, $\mathbf{C}$, $\mathbf{S}$
1: **Initialization:** $\hat{\theta}_{0|0} \leftarrow (-12)$, $\hat{\mathbf{P}}_{0|0} \leftarrow \mathbf{I}_4$
2: **for** $t = 1, \dots, T - 1$ **do**
3: Find χ_i , $w_i^{(m)}$ and $w_i^{(c)}$ according to (22)
4: $\hat{\chi}_i \leftarrow (23)$
5: **Prediction:**
6: $\hat{\theta}_{t|t-1} \leftarrow (24)$
 $\hat{\mathbf{P}}_{t|t-1} \leftarrow (24)$
7: **Update:**
8: Find $\hat{\mathbf{z}}_{t|t-1}$, $\hat{\mathbf{P}}_t^{\text{ZZ}}$ and $\hat{\mathbf{P}}_t^{\Theta Z}$ according to (25)
9: $\mathbf{K}_t \leftarrow \hat{\mathbf{P}}_t^{\Theta Z} (\hat{\mathbf{P}}_t^{\text{ZZ}} + \mathbf{C})^{-1}$
10: $\hat{\theta}_{t|t} \leftarrow (26)$
 $\hat{\mathbf{P}}_{t|t} \leftarrow (26)$
11: **end for**

4.5. Particle filter

Similar as the UKF, the PF approximates the posterior PDF of the state with sample points, called particles, but with essential difference that these particles are selected at random. Essentially, it is nothing else but an ordinary randomization technique whose performance and computational complexity are directly proportional to the number of particles used [2,4]. The particles are iteratively updated according to new observations, and no linearity nor Gaussianity assumptions are required [24,25].

First, at t , N_p particles are randomly generated, $\mathbf{p}_t^n$, according to the transition prior

$$\hat{\mathbf{p}}_{t|t-1}^n \sim \mathcal{N}(\mathbf{S} \hat{\mathbf{p}}_{t-1|t-1}^n, \mathbf{Q}), \quad n = 1, \dots, N_p. \quad (27)$$

Then, the likelihood that each particle corresponds to a given observations is evaluated, i.e.,

$$\omega_t^n = \omega_{t-1}^n p(\mathbf{z}_t | \hat{\mathbf{p}}_{t-1}^n). \quad (28)$$

In this process, some particles might be given very low importance weights, ω_t^n , for them to be negligible. Hence, these particles are discarded, and the samples with big weights are duplicated. This technique is known as resampling method [24,25], and is very common for PF.

After resampling, the weights are normalized, so that they sum up to 1. The state update is calculated as the mean of the posterior, by using the normalized weights, $\tilde{\omega}_t^n$, and resampled particles, $\hat{\mathbf{p}}_{t|t}^n$, i.e.,

$$\hat{\theta}_{t|t} = \frac{1}{N_p} \sum_{n=1}^{N_p} \tilde{\omega}_t^n \hat{\mathbf{p}}_{t|t}^n. \quad (29)$$

The step by step proposed PF algorithm is outlined in Algorithm 5.

Algorithm 5 PF Algorithm Description

Require: $\mathbf{z}_t$, for $t = 0, \dots, T - 1$, $\mathbf{Q}$, $\mathbf{C}$, $\mathbf{S}$, N_p
1: **Initialization:** $\hat{\theta}_{0|0} \leftarrow (-12)$, $\hat{\mathbf{P}}_{0|0} \leftarrow 5\mathbf{I}_4$, $\hat{\mathbf{p}}_{0|0}^n \sim \mathcal{N}(\hat{\theta}_{0|0}, \hat{\mathbf{P}}_{0|0})$, $\omega_0^n = \frac{1}{N_p}$,
 $n = 1, \dots, N_p$
2: **for** $t = 1, \dots, T - 1$ **do**
3: **Prediction:**
4: $\hat{\mathbf{p}}_{t|t-1}^n \leftarrow (27)$
5: **Update:**
6: $\omega_t^n \leftarrow (28)$
7: $\hat{\mathbf{p}}_{t|t}^n \leftarrow$ resample if necessary
8: Normalize the weights: $\tilde{\omega}_t^n = \frac{\omega_t^n}{\sum_{n=1}^{N_p} \omega_t^n}$
9: $\hat{\theta}_{t|t} \leftarrow (29)$
10: **end for**

5. Performance results

The validity of the proposed algorithms is evaluated through computer simulations. Two essentially different scenarios are considered, in which the target takes sharp maneuvers and a more smooth one. The target state transition follows the state model (3), and at each time instant the radio measurements are generated according to (4). In both scenarios, the reference distance is set to $d_0 = 1$ m, the reference power to $P_0 = -10$ dBm, the path loss exponent to $\gamma = 3$, and $N = 3$ anchors are fixed at $[[70, 10]^T, [40, 70]^T, [10, 40]^T]$. A sample is taken every $\Delta = 1$ s during $T = 150$ s trajectory duration in each Monte Carlo, $M_c = 1000$, run. Furthermore, $\sigma_{n_i} = 9$ dB, $\sigma_{m_i} = 4$ degrees, and $q = 2.5 \times 10^{-3} \text{ m}^2/\text{s}^3$. For the proposed PF algorithm, $N_p = 200$ is used, and the tuning factor is set to $q = 2.5 \times 10^{-1} \text{ m}^2/\text{s}^3$ in order to allow greater search space to the particles. The performance metric used here is the root mean square error (RMSE), defined as

(a) Via KF, UKF, WLS, and KF [4].

(b) Via MAP, EKF and PF.

Fig. 2. The true target trajectory in scenario 1, compared with the estimated trajectory.

$RMSE_t = \sqrt{\sum_{i=1}^{M_c} \frac{\|x_{i,t} - \hat{x}_{i,t}\|^2}{M_c}}$, where $\hat{x}_{i,t}$ denotes the estimate of the true target location, $x_{i,t}$, in the i th M_c run at time instant t .

The proposed algorithms described in Section 4 are compared with the KF [4], where the initial target state was estimated by solving the LS method used in [4] to linearize the observation model⁵, and the sequential WLS localization method [15] applied here to linearize the observation model, which does not take advantage of the prior knowledge.

The first scenario is illustrated in Fig. 2, where the initial target location is at $[21, 20]^T$. From the figure, one can observe that all algorithms, except the WLS one, perform fairly good in the first phase ($t \leq 40$) of the target trajectory. The reason for this behavior is that the trajectory is relatively smooth at this phase, and all algorithms benefit from the prior knowledge, except the WLS which is based purely on the quality of the observations. Furthermore, Fig. 2 exhibits a slight performance deterioration of all algorithms after each sharp maneuver of the target, especially as the target approaches any of the anchors. This is somewhat expected since the sudden turn of the target diminishes the prior knowledge stipulated by all tracking algorithms, and the proximity of an anchor intensifies the importance of that particular observation in comparison with the other ones. However, it can be seen that these impairments are mild, and that all tracking algorithms quickly recover from them.

The second scenario is illustrated in Fig. 3, where the initial target location is at $[35, 15]^T$. It can be seen that, when the target constantly changes its direction, but without particularly sharp maneuvers, the performance of the tracking algorithm betters in

(a) Via KF, UKF, WLS, and KF [4].

(b) Via MAP, EKF and PF.

Fig. 3. The true target trajectory in scenario 2, compared with the estimated trajectory.

Fig. 4. RMSE versus t (s) comparison, when $\sigma_{n_i} = 9$ dB, $\sigma_{m_i} = 4$ degrees, $\gamma = 3$, $P_0 = -10$ dBm, $d_0 = 1$ m, $q = 2.5 \times 10^{-3} \text{ m}^2/\text{s}^3$, $M_c = 1000$.

general. Once again, modest performance deteriorations are observed in the proximity of any anchor.

Fig. 4 illustrates the RMSE versus t comparison in the first considered scenario. The figure exhibits that the tracking algorithms, in general, perform better than the sequential localization one, as expected, only at the critical points is where the localization algorithm possibly has advantage in comparison with the tracking ones. Moreover, Fig. 4 shows that the PF has the most stable performance, while the proposed KF outperforms the one in [4], suggesting that the new linearization technique offers considerable gain.

Fig. 5 illustrates the RMSE versus t comparison in the second considered scenario. From this figure, it is absolutely clear that combining the prior knowledge with observations can significantly improve the estimation accuracy. This observation is in line with

⁵ In order to make the comparison fair, the true value of the PLE is given to the KF [4] at every time step.

Fig. 5. RMSE versus t (s) comparison, when $\sigma_{n_i} = 9$ dB, $\sigma_{m_i} = 4$ degrees, $\gamma = 3$, $P_0 = -10$ dBm, $d_0 = 1$ m, $q = 2.5 \times 10^{-3} \text{ m}^2/\text{s}^3$, $M_c = 1000$.

Table 1
Average RMSE (m) of the considered algorithms.

Algorithm	Scenario 1	Scenario 2
MAP	2.86	2.96
KF	3.12	3.20
EKF	2.80	2.09
UKF	2.78	2.10
PF, $q = 2.5 \times 10^{-1} \text{ m}^2/\text{s}^3$	2.60	2.62
PF, $q = 2.5 \times 10^{-3} \text{ m}^2/\text{s}^3$	37.41	31.98
KF [4]	4.22	4.30
WLS	4.31	4.14

expectations, since the target trajectory is fairly smooth. Fig. 5 also exposes the proposed EKF and UKF algorithms as the best options for this scenario, and confirms the superiority of the proposed KF over the one in [4].

Finally, the average RMSE performance of the considered algorithms for both scenarios is presented in Table 1. In order to show the dependence of the PF on the tuning factor q , the results of PF for two different values of q are presented. Obviously, the PF is highly reliant on q , such that higher values of q favor its performance. This is not completely surprising, since higher values of the tuning factor allow greater search space for particle deployment. From the table, we can also observe that the proposed KF outperforms the KF in [4] for about 1 m. Furthermore, it can be seen that the proposed EKF and UKF have similar performance, and that, unlike the other algorithms, they significantly improve their performance in the second scenario, in comparison with the first one. This behavior is somewhat anticipated for EKF, which is a derivative-based estimation technique; thus, naturally favors mild target trajectories. In the case of UKF, although the mean and covariance are accurately captured with the discrete distribution with finite samples up to the 3rd order terms, its performance still depends on the higher order terms, and it is known to be scenario-dependent⁶ in general [22,23].

It is also worth mentioning that, in terms of computational complexity, the most expensive operations of an algorithm are matrix multiplication and inversion [26]. These operations are a part of practically all considered algorithms (except for the PF, but this algorithm has other drawbacks related with the number of

Table 2

Running time (s) of the considered algorithms for $M_c = 1000$. Processor: Intel(R)Core(TM)i7-4710HQ, CPU@ 2.50 GHz.

Algorithm	MAP	KF	EKF	UKF	PF	WLS	KF [4]
Scenario 1	36.0	31.3	37.0	54.1	467.6	28.8	31.2
Scenario 2	36.1	31.4	37.1	55.2	468.9	28.8	31.9

particles used), and it can be said that essentially all considered algorithms are of the same order of complexity. Nevertheless, the UKF and the PF use a number of sigma points and particles respectively, which affect their computational complexities and consequently their execution times, as it is shown in Table 2.

In Table 2, the running times (s) of the considered algorithms for both scenarios for $M_c = 1000$ are presented. From it, one can see that the PF requires the longest running time of all algorithms, which is one order of magnitude longer than the running time of the remaining algorithms. This is not surprising, since the PF algorithm uses a large number of particles, which significantly increases its computational load. Similarly, the running time of the UKF is the second longest, which is a result of using sigma points in its execution. The running time of the remaining algorithms is practically the same. However it is worth pointing out that the EKF is a derivative-based algorithm, and finding the derivative can be very difficult or even not possible in some cases.

6. Conclusions and future work

In this work, the target tracking problem that utilizes hybrid (RSS and AoA) measurements was addressed. We have presented a novel *linearization* technique to approximate the highly non-linear measurement model into a linear one. Extensive study of the problem based on Bayesian methodology has been carried out, which resulted in the proposal of two novel tracking algorithms, namely MAP and KF. Moreover, to the best of our knowledge, the application of the EKF and UKF algorithms for efficient solving of the considered RSS-AoA target tracking problem was presented here for the first time. Furthermore, the well-known PF algorithm was also presented. All of the presented algorithms take advantage of the prior knowledge, extracted from the state model, which is complemented with the observation information in order to enhance the estimation accuracy. The new algorithms were compared with the existing KF algorithm and the classical localization algorithm that neglects the prior knowledge in two different scenarios: where the target takes sharp maneuvers and a more smooth one. The simulation results have confirmed that employing the prior knowledge into an estimator can significantly reduce the estimation error. Moreover, the simulation results have confirmed that the proposed algorithms efficiently solve the target tracking problem, and that the proposed *linearization* technique used in the new KF offers considerably better estimation accuracy in comparison with the existing one in [4].

It should be pointed out that this work considered the tracking problem where the anchors were assumed static. A different approach in which the anchors are mobile and move in a certain direction based on pre-established rules that take into account the target estimate is left for future work. In this way, we hope to further improve the estimation accuracy of our algorithms and possibly reduce the number of anchors required for efficient problem solving.

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⁶ Note that the EKF algorithm might not be well approximated by the first-order Taylor series expansion in the case of error propagation [22,23], and that the UKF algorithm depends on some heuristically chosen values (e.g., the parameter κ in [20] which scales the sigma points of the unscented transformation towards or away from the mean of the prior distribution). Thus, the two algorithms might not converge always, as was the case in the considered scenarios. However, the results presented here are all for $M_c = 1000$ cases in which they converged.

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