Research Article

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Economic Disposal Quantity of Leftovers kept in storage: a Monte Carlo simulation method

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Abstract: This article describes how to reach an item's threshold, or in other words, the limit time for it to be retrieved from stock and sold for a different use, as well as the remaining foreseen period for this situation to occur. Once a minimum length, or weight, is reached, left quantities are more difficult to sell, as demand often exceeds the remaining parts or leftovers. The number of unfulfilled orders increases, as time goes by, until it becomes further cost effective to dispose the leftover and sell it for a lower price and alternative use. A Monte Carlo simulation model was built in order to consider the randomness of future transactions and quantifying consequences providing this way a simple and effective decision-making framework.

Keywords: Decision-making; Economical Optimization; Monte-Carlo Simulation; Stochastic Process

1 Introduction

In the retail trade activity, a particular situation might occur when a piece of a material in the form of a reel (*e.g.* electric cable, flexible tube, rope, wire, paper, or tape) or in the form of a rod or rigid tube is cut into different lengths to satisfy custom orders. Once reduced lengths become more difficult to sell, only entire pieces are acceptable by the end user, since joints are technically and/or economically unfeasible [1].

This type of subject can be better evaluated using simulation techniques. Which concerns Monte Carlo simulation there are several recent developments in literature that can be cited, *e.g.* articles on stochastic approach for dynamic of pricing [2–4], and papers about new approaches for random variable and process characterization [5–7].

Chen and Simchi-Levi (2004) analyzed a model of periodic review, unique product and infinite horizon, in which pricing decisions and production / inventory are simultaneously taken. The demands at different periods are randomly distributed variables that are independent of each other and their distributions depend on the price of the product. Price and order decisions are made at the beginning of each period and shortages are backlogged. The order cost also includes a fixed price and a variable cost proportional to the quantity ordered. The goal is to maximize the expected discount, or expected average, and profit over the infinite planning horizon. It has been shown that a stationary policy is optimal for both discounted and average profit models with general demand functions. In such policy, the stocking period is managed based on a classical (s, S) policy and the price is determined based on the stock position at the beginning of each period [2].

Chen, Ray and Song (2006) studied a problem of pricing and control of periodic inventory for a retailer, which faces stochastic demand prices, under very general modeling assumptions. Any unsatisfied demand is lost and any remaining inventory at the end of the finite sales horizon has a residual value. The cost component for the retailer includes exploration, scarcity, and both variable and fixed order costs. The retailer's goal is to maximize its expected discounted profit over the sales horizon by dynamically deciding on optimal pricing and replacement policy for each period. Authors showed that, under an assumed additive demand function, at the beginning of each period, a (s, S) policy is optimal for replenishment, and the price value depends on the stock level after the replenishment decision. Their numerical study also suggested that for a sufficiently long sales horizon, optimal policy is almost stationary. In addition, the fixed order cost plays a role in authors' modeling framework. On the other hand, the impact on profit of dynamically changing the retail price, as opposed to a single fixed price across the entire sales horizon, also increases with the fixed cost of ordering [3].

Levin, McGill and Nediak (2012) have presented a dynamic pricing model for oligopolistic firms that sell differ-

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entiated consumable products to multiple finite segments of strategic consumers who are aware that price is dynamic and can delay their purchases accordingly. This model encompasses the strategic behavior of both firms and consumers in a dynamic stochastic game in which the objective of each firm is to maximize its expected total revenues, and each consumer responds according to a consumer choice model of intensity purchasing allocation. The model provides awareness about equilibrium price dynamics under different levels of competition, asymmetry between firms and multiple market segments with variable properties. Authors have shown that strategic consumer behavior can have serious revenue impacts if companies ignore this behavior in their dynamic pricing policies. In addition, optimal balance responses to consumer strategic behavior can recover only part of the lost revenue. A key conclusion is that companies can benefit more from limiting the information available to consumers than from allowing full information and responding to strategic behavior in an ideal way [4].

Additionally, Paola and Pinnola (2012) discussed the probabilistic representation of the probability density function (PDF) or the characteristic function (CF) in terms of complex fractional moments, showing that such complex moments are related to Riesz and complementary integrals of Riesz at the origin. According to the authors, discretization leads to the conclusion that with a few fractional moments, the entire PDF or CF can be restored.

Also, Alotta, Paola and Pinnola (2017) presented an approach that characterizes normal processes of multivariate stochastic vectors, based on the evaluation of complex spectral moments. The knowledge of these moments allows to obtain the power spectral densities providing a complete characterization of the processes of multivariate stochastic vectors [6]. These quantities are the generalization of the full-order spectral moments introduced by Vanmarcke (1971) [7]. For more general pioneered work see the contributions proposed by Arrow (1970, 1974) [8, 9].

Having reviewed some literature regarding the issue, we propose the following research objective:

> *The construct and use of a Monte Carlo simulation model for decision-making of randomness future transactions and quantifying costs.*

In other words, one intends to understand what the limit time for an item is, to be retrieved from stock and sold for a different use, as well as the remaining foreseen period for this situation to occur.

At this point, two courses of action are possible: (i) sell immediately the remaining piece for an alternative use (*e.g.* for scrap) at a lower price than the market one; or (ii) wait in the expectation to sell it at the market price, despite uncertain it might be.

From an economical perspective, there must be a minimum length (or weight), where the expected marginal benefit from selling to the every-day market turns to be smaller than the expected marginal benefit from selling to an alternative use [10, 11].

This so called minimum or 'optimal economic equilibrium quantity', can be deduced by simulating the upcoming future, and assuming the following defined condition proposed earlier by Assis and Figueira [1]:

$$ERM < ERA$$

Where, *ERM* is the **Expected Revenue** from the **Market** and *ERA* is the **Expected Revenue** from an **Alternative** use.

2 Methods and Tools

To deal with the current problem an algorithm was developed by the authors using the constant price [1, 10]. As so, one defines the following:

2.1 Opportunity for ERM sales

The economic potential / mathematical expectation of sales - or the opportunity for *ERM* sales - at any time t (month, week, day), as an alternative to the *ERA* divestment opportunity at the same time t, can be calculated by summing all expected sales (weighted with probabilities) in time interval T-t, where T is the instance at which sales probability equals 0. This result can be expressed by the following Expression [1, 10]:

$$ERM_{T-t} = \sum_{t}^{T} P_t, \overline{Q}_t, v_t(1+i)^{-t} \qquad (1)$$

Where,
P_t – Probability of selling the existing length at the end of period t;
$\overline{Q}_t$ – Expected average sales rate during each period t;
v_t – Net selling price supposed to remain valid during each period t;
i – Stock holding cost;
T – Analysis ending time (after which $P_t = 0$).

Making note that in previous Expression (1), the sum of expected (or potential) sales is discounted, making possible to compare ERM_{T-t} and ERA_t at any time t.

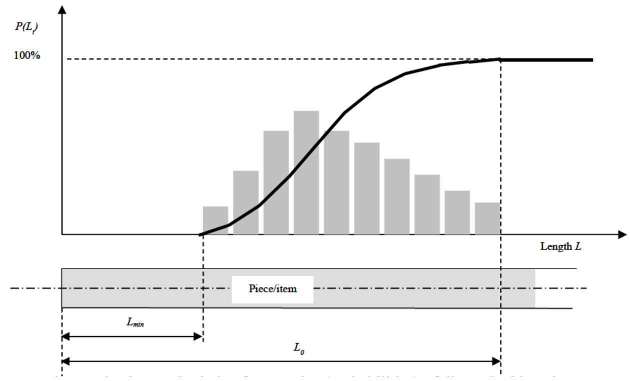

Figure 1: Absolute and relative frequencies (probabilities) of dispatched lengths.

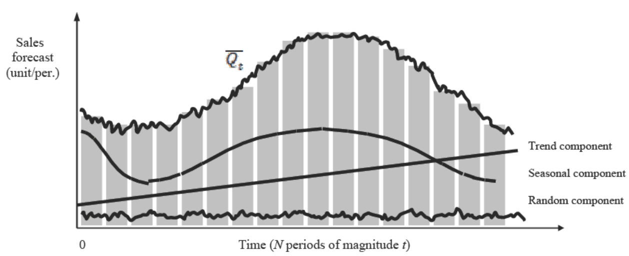

Figure 2: Evolution of expected average sales profiles within each period *t* over the time period of analysis.

The stock holding cost *i* is equal to the sum of the opportunity cost of tied up capital with warehousing costs [1, 10].

It is assumed that the frequencies of the dispatched lengths in recent past are known and remain constant over near future, according to Figure 1.

In this figure, L_0 represents the maximum length ever delivered for an order fulfillment. As it can be seen, the probability (or success) of a sale is maximum - equal to 1 - until the piece reaches the length L_0, *i.e.* for $L \geq L_0$.

From here on, the probability decreases progressively till 0, which occurs when the length of the piece reaches L_{min}.

The average sales $\overline{Q}_t$, during each period *t*, can assume any value according to the forecasted sales profile within a time period of analysis and may consider the existence of eventual trend, seasonality, and random as depicted in next Figure 2.

Being likely the net price, v_t sales might suffer a major change over different periods of analysis and the *current price* should be adopted instead. This will not change any calculation steps but contributes to increase complexity. In order to keep the case as simple as possible, the net selling price, v_t is assumed constant and equal to *v*. Thus,

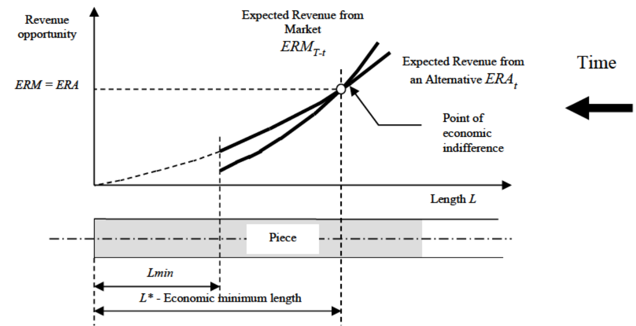

Figure 3: Variation of ERM_{T-t} vs. ERA_t with remaining length or over time.

previous Expression (1) can be rewritten as follows:

$$ERM_{T-\tau} = v, \sum_{t}^{T} P_t, \overline{Q}_t, (1+i)^{-t} \qquad (2)$$

2.2 Opportunity for ERA sales

As an alternative, disposal for scrap or any other possible solution, might occur at the end of any time period *t*. The economic potential or the opportunity for disposal ERA_t is calculated by the next Expression [1, 10]:

$$ERA_t = s.L_t.(1+j)^{-t} \qquad (3)$$

Where:

s – Unit sales price (for scrap or any other solution);
L_t – Existing length at the end of period t;
j – Opportunity cost of tied up capital.

In Expression (3), the product $s.L_t$ is discounted, being possible to compare ERM_{T-t} and ERA_t at any period *t*. The discount rate *j* is, for this case, equal to the cost of capital held up, an opportunity cost.

2.3 Trade-off point

A trade-off point can be established with Expressions (2) and (3). This is graphically depicted in the following Figure 3.

As it can be observed, the later the alienation is decided, the lower *L* is, as well as values for ERM_{T-t} and ERA_t. While $L > L_0$ is met, the probability of satisfying any request is 1. As soon as $L < L_0$, the probability decreases progressively, until it reaches the value of 0 for $L = Lmin$, thus becoming a leftover. There is, however, a certain point where the two curves intercept, *i.e.*, a point of economic indifference. This point corresponds to length L^*, which

Table 1: Frequency of quantities supplied by order.*

Lower limit range (kg/order)	Upper limit range (kg/order)	Frequencies (%)	Cumulative frequencies (%)
0	15	5	5
15	20	10	15
20	25	20	35
25	30	35	70
30	35	25	95
35	40	5	100

* the average value for this frequency distribution is 26.25 kg/order.

Table 2: Frequency of weekly orders.

Number of share/weeks	Frequencies (%)	Cumulative frequencies (%)
(average of week t) - 2	5	5
(average of week t) - 1	25	30
(average of week t) *	30	60
(average of week t) + 1	25	85
(average of week t) + 2	15	100

*closest multiple quotient: demand in week *t*/26.25

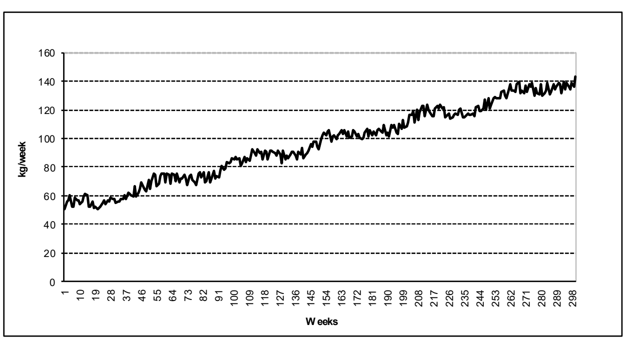

Figure 4: A run of the simulation model showing future orders over 300 weeks with trend, seasonality and randomness.

represents an economic minimum length, and can be addressed as a point of change in the decision or *economic disposal quantity.* In fact:

– for $L_t > L^{*}$, is more cost effective to keep the part, continuing the market sale as $ERM_{T-t} > ERA_t$; and
– for $L_t < L^{*}$, an immediate disposal becomes more cost effective as $ERM_{T-t} < ERA_t$.

The value for L^{*} can be deduced analytically by equalizing Expressions (2) and (3) and solving the following Equation for L_t [1],

$$v.\sum_{t}^{T} P_t, \overline{Q}_t, (1+i)^{-t} = s.L_t, (1+j)^{-t} \qquad (4)$$

Being manual calculations laborious, Monte-Carlo simulation method presents a valuable alternative [12, 13]. An example of application illustrates the use of the developed algorithm.

3 Results

A company trades alloy steels for the metal-mechanical industry in the form of rods, cut to different lengths. These rods, when new, have a standard length. When a rod reaches a minimum length, it becomes progressively more difficult to fulfill orders and to judge how long it should be retained before being sold for alternative use.

Let one assume a piece with 1,000 kg. This material is traded in the market at 5 €/kg, being sold immediately for an alternative use at 3 €/kg. Considering an opportunity cost of the capital to be held in stock equal to 15% and 20% as a warehouse costs, both per year, the following question arises: *what is the minimal weight (to be sold) for an alternative use?*

To answer this question, one needs to investigate sales history. Analyzing the sales of the last two years, it was possible to set a season component, an increasing trend component, and a random component. Then, it is also necessary to define, as a frequency, the amount of received orders and their respective size. Hence, Table 1 and 2 show an analysis of the last two months (since the rod's cross section is constant, it is rather indifferent to use linear or mass units).

In order to solve this problem, a representative-simulation model was developed in MS-Excel©. In this simulation model, the time variable can progress in two ways: (1) by *variable increments* (for each moment of an order arrival), or (2) by *fixed increments* (days, weeks, etc.). In the last case, for each period, a random quantity of orders, as well as a random quantity (number of kg) for each order was simulated. The second alternative was chosen regarding simplicity.

This simulation model allowed ones to fix the difference between ERM_{T-t} and ERA_t at the end of each period t and revealing at which time t^{*} (or length L^{*}) this difference is worthless. Denote that the parameters that characterized demand were trend, seasonality, and randomness, along with a time horizon of 300 weeks. Also, fixed data corresponded to selling price, disposal price, opportunity

Table 3: Simulated demand obtained in one specific run of the simulation model.

Week (1)	Simulated demand (2)	Average number of delivers (3)	Simulated number of delivers (4)	1st deliver (5)	2nd deliver (6)	3rd deliver (7)	Total of delivers (8)
10	54.37	2	2	28.304	21.573	-	49.877
11	60.501	2	2	27.194	21.451	-	48.645
12	54.821	2	2	21.486	33.748	-	55.235
13	60.846	2	3	28.346	33.821	34.211	96.378
14	56.082	2	2	33.908	27.623	-	61.531
15	54.104	2	2	34.199	39.981	-	74.180
16	58.297	2	1	27.866	-	-	27.866
17	52.379	2	1	21.081	-	-	21.081
18	51.531	2	3	20.809	15.280	34.009	70.098
19	55.210	2	2	27.876	27.592	-	55.469
20	59.363	2	1	21.644	-	-	21.644
21	57.302	2	2	27.520	27.252	-	54.773
22	57.230	2	3	21.526	21.128	15.380	58.034
23	59.561	2	3	15.483	28.116	27.624	71.223
24	50.939	2	3	27.707	21.382	34.541	83.631
25	58.022	2	2	0.643	0.597	-	1.240
26	51.207	2	2	28.062	34.096	-	62.157
27	53.655	2	3	27.622	27.452	33.600	88.673
28	53.389	2	2	15.614	15.391	-	31.006
29	59.091	2	2	27.588	33.809	-	61.396
30	55.791	2	2	27.855	27.000	-	54.855

capital cost, storage cost, and quantity presently existing in stock.

The simulation model randomly generated, at each run, a demand profile over 300 weeks. Next Figure 4 beneath shows one of these profiles.

Subsequently data from Table 1 and 2 along with the desired number of runs were set. One should note that the number of runs depends on the desired outputs' accuracy. A confidence level of 90% was set. After 500 runs, the simulation model provided the following statistics:

- Economic Disposal Quantity = 5.97 ± 0.37 kg with a sampling error of 6.2 %;
- Present value of *ERM* = 4,698 ± 1.79 € with a sampling error of 0.04%;
- Present value of *ERA* = 3,000 €; and
- Remaining time until L^* is reached = 30 ± 1.51 weeks with a sampling error of 4.98%.

As matter of fact, for the existing quantity of 1,000 kg, the following results were found:

- *ERM* = 4,698 €, and
- *ERA* = 3,000 €.

Once *ERM* > *ERA* (or 1,000 kg > 5.97 kg) the remaining piece should be retained, as *ERM* is far higher than *ERA*. In addition, the simulation model also provided a 30 week-long estimate as time span left, until the moment for disposal arrives. Following Table 3 and 4 depict results from one of the numerous runs performed by the simulation model, *i.e.*:

- Table 3 shows the orders profile from week 10 through week 30, based on the parameters and the frequency distributions from Table 1 and 2; and
- Table 4 shows the level of stock and values of *ERM* sales and *ERA* disposal, verified at the end of weeks 10 to 30.

Column (3) was obtained by the ratio between column (1) / 26.25, where 26.25 is the mean value of each order delivered (see Table 1). The result was rounded to the nearest multiple. Column (4) was obtained by interpolating the frequency distribution of the weekly number of delivers represented on Table (2), by Monte-Carlo simulation, taking as mean values those in column (3). Columns (5), (6) and (7) were obtained by interpolating the frequency distribution of the delivered quantities shown in Table 1 and according

Table 4: Snap values resulting from one of the simulator iterations

Week (1)	Initial stock (9)	Foreseen orders (kg) (10)*	ERM_{T-t} in the week (€) (11)	ERM_{T-t} discounted (€) (12)	ERM_{T-t} discounted and accumulated (€) (13)	ERA_t in the week (€) (14)	ERA_t discounted (€) (15)
10	515.406	49.877	249.385	235.400	2347.186	1546.217	1505.212
11	465.529	48.645	243.225	228.264	2111.786	1396.586	1355.900
12	416.884	55.235	276.174	257.695	1883.522	1250.651	1210.957
13	361.649	96.378	481.891	447.059	1625.826	1084.946	1047.692
14	265.271	61.531	307.655	283.775	1178.767	795.812	766.423
15	203.740	74.180	370.899	340.142	894.992	611.219	587.067
16	129.560	27.866	139.330	127.040	554.850	388.679	372.319
17	101.694	21.081	105.406	95.556	427.810	305.081	291.455
18	80.613	70.098	350.491	315.908	332.254	241.838	230.416
19	10.514	0.000	0.000	0.000	16.346	31.543	29.973
20	10.514	0.000	0.000	0.000	16.346	31.543	29.892
21	10.514	0.000	0.000	0.000	16.346	31.543	29.812
22	10.514	0.000	0.000	0.000	16.346	31.543	29.732
23	10.514	0.000	0.000	0.000	16.346	31.543	29.652
24	10.514	0.000	0.000	0.000	16.346	31.543	29.573
25	10.514	1.240	6.201	5.368	16.346	31.543	29.493
26	9.274	0.000	0.000	0.000	10.978	27.822	25.945
27	9.274	0.000	0.000	0.000	10.978	27.822	25.875
28	9.274	0.000	0.000	0.000	10.978	27.822	25.806
29	9.274	0.000	0.000	0.000	10.978	27.822	25.736
30	9.274	0.000	0.000	0.000	10.978	27.822	25.667

* = column (8) of Table 3

to Monte-Carlo simulation. Column (8) is equal to the sum of columns (5), (6) and (7).

Comparing Table 4 to Table 3, one can notice that in weeks 10 to 30, many orders (columns (5), (6) and (7) of Table 3), cannot be satisfied due to insufficient stock. In fact, delivers are satisfied until week 19, where L^* is attained. More precisely, in week 18, *ERM* (332.254) > *ERA* (230.416) and in week 19, *ERM* (16.346) < *ERA* (29.973). For this case, the economic disposal quantity is found to be approximately 10.514 kg, being reached in week 19. In the appendix of this paper a print screen of the constructed Monte Carlo model is presented.

4 Discussion and Conclusion

The used calculation algorithm provided the threshold quantity of any piece of material when it became costlier effective to sell it, for an alternative use, at a lower price. This quantity, compared with the existing balance, whenever a transaction takes place, provides a simple and effective way for decision-making, as established previously as a research objective of this paper. It is also important to notice that the described analysis can be easily implemented on a computer, automatically triggering a warning, as soon as the threshold is reached. In this case, the variables-based decision must be updated on a time basis or whenever a change in data occurs, such as the number of transactions of items in stock. Finally, and to increase the level-headedness of the present simulation model, one is in condition to access for example forecast values of $\overline{Q}_T$, once the model is properly connected to a dedicated-app for past-sales and future prediction analyses.

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Appendix : Print screen of the Monte Carlo model (in Portuguese).